Theoretical Determination of Forming Limit Diagram for Steel ,Brass and Aluminum alloy sheets

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Abstract :

Sheet metal forming is defined as the ability of metal to deform plastically (deformation by Stretching or drawing) or changing the shape of the sheet into a new desirable shape with out necking or crack . To control the operation of sheet metal forming with out failure. A diagram is used in which the range accepted , failure and critical deformation range are shown . This diagram is known as the Forming limit diagram. It is considered as one of the important tool to determine the formability of sheet metals. Every sheet metal has its own forming limit diagram which determines its formability, strain limit and the forming regions. These diagrams can be assessed using theoretical and experimental approaches, In this paper, the FLD is determined using different yield criteria Hill1948, Hosford1979 and modified Hosford 1985. It is shown that the determination of forming limit curve using the modified Hosford 1985 criterion with the (M-K) analysis , gave the best results compared with the other used criteria .Using this criterion gave the closest forming limit curve to that obtained experimentally, but with different criterion index for different alloy . The value of the index (a=6) gave the best results for brass, while (a=8) gave the best results for aluminum alloy and mild steel.

التعيين النظري لمنحني حد التشكيل لصفائح الصلب والبراص والألمنيوم

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الخلاصة

عمليات تشكيل الصفائح المعدنية تعرف على أنها قابلية المعدن على تحمل التشويه اللدن (التشكيل بالمط أو السحب) أي تغيير شكل الصفيحة الى شكل مرغوب فيه دون حدوث تخصر **أو كسر** ، ولكي نسيطر على عملية تشكيل الصفائح دون حدوث فشل يتم استخدام مخطط يبين فيه مناطق التشكيل المقبولة ،ومناطق الفشل ،والمناطق الحرجة ، وهو مخطط حد التشكيل , يعتبر

مخطط حد التشكيل من الأدوات والوسائل المهمة في تحديد قابلية تشكيل الصفائح المعدنيـة . لكل صفيحة معدنية منحني حد تشكيل خاص بها يحدد قابليتها على التشكيل وحدود الانفعال ومنـاطق التشـكيل .فـي هـذا البحـث تـم تعيـين منحنــي حـد التشـكيل نظريـا باسـتخدام ثـلاث نظريـات للخضوع(Hill1948)، (Hosford1979)، (Hosford1985) ومقارنته مع المنحنـي المعين عمليا.

فـي الجانـب النظـري وجـد أن تعيـين منحنــي حـد التشـكيل باسـتخدام نظريـة الخضـوع (Hosford 1985) المطـورة مـع تحليـل (M-K) أعطـت أفضـل النتـائج مقارنـة بـالنظريتين الأخـرتين مـن ناحيـة قـرب المنحنـي مـن ذلـك الـذي تـم تعيينـه عمليـا ولكـن بـاختلاف أس نظريـة الخضوع لكل من المعادن المستخدمة ، حيث أنـه عند استخدام الأس (a=6) كـان هـو الأفضـل للبراص ، بينما عند استخدام الأس (a=8) كان الأفضل بالنسبة للألمنيوم والصلب .

Notation

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$\sigma_1,\sigma_2,\sigma_3$	Principle stresses
$\varepsilon_1\ \varepsilon_2\ \varepsilon_3$	Principle strains
m	Strain rate sensitivity
n	Strain hardening exponent
σ'	effective stress
$\acute{\varepsilon}$	effective strain
$\dot{\varepsilon}$	strain rate
$\dot{\varepsilon}'$	effective strain rate
ρ	ratio of minor strain to major strain
ta	thickness of the sheet
tb	Thickness of groove
α	Principle stress ratio
f	Imperfection factor
a	Yield criterion index
K	Strength coefficient
R´	Normal plastic anisotropic ratio
R1,R0	Plastic anisotropic ratio with rolling direction

R2,R90	Plastic anisotropic ratio transverse to rolling direction
φ	ratio of principle stress to effective stress
β	Ratio of effective strain to principle strain
M-K	Marciniak-Kuczynski analysis

1.Introduction

Forming processes are among the most important metal working operations. The industrial process of sheet-metal forming is strongly dependent on numerous interactive variables such as material behavior, lubrication, forming equipment, etc.

Forming limit Diagram is a representation of the critical combination of the two principal surface strains major and minor above which localized necking instability is observed. Forming limit curve (FLC) provides excellent guidelines for adjusting material, tooling and lubrication conditions. Also it is strongly dependent on material parameters, The idea of forming limit diagrams was first introduced by Keeler [1], when he observed that the maximum local elongation was not enough to determine the possible straining rate of a sheet .He established that the plotting of the principal strains at fracture $\varepsilon 1$, and $\varepsilon 2$ on two axes of a same diagram gave a curve : the forming limit curve. This curve, first restricted on the area $\varepsilon 2>0$, was made complete for $\varepsilon 2< 0$ by Goodwin's works [2].This curve is interesting because it divides the plane into two zones .The success area under the forming limit curve and the fail area above it, for a deep drawing operation .The criteria to reject the drawn parts is now the onset of localized necking.

Hill [3] was the first who proposed a general criterion for localized necking in thin sheets under plane stress states. His analysis predicts localized plastic deformation in the negative minor strain region. Marciniak and Kuckzinsky (M–K) [4] have proposed the first realistic mathematical model for theoretical determination of FLDs that suppose an infinite sheet metal containing a region of local imperfection where heterogeneous plastic flow develops and localizes. Hutchinson and Neale

[5] extended M–K theory using a J2 deformation theory. Therefore, the left and right hand sides of the forming limit diagram can be calculated by M–K analysis.

Sheet metals exhibit a highly anisotropic material behavior by cold rolling. It is therefore of major importance to extend the plastic instability analysis to anisotropic materials. Constitutive relations for the plastic yielding and deformation of anisotropic metals at a macroscopic level were proposed long ago by Hill 1948[6] This theory was the simplest conceivable one for anisotropic materials, however, inevitable limitations of its range of validity have eventually became apparent. The original M-K analysis [4] was based on Hill's 1948 yield criterion [6]. However , it can be seen from the comparison with experiments and predicted results of Painter and Pearce 1974[7] that this analysis overestimates the limit strains towards the equibiaxial strain region, and underestimates the limit strains towards the plane strain region, particularly for materials with R values less than unity such as aluminum or brass . In addition, the calculated limit strains for the right hand side of the FLDs are very sensitive to the material anisotropy, a phenomenon that has not been observed in experiment .Sowerby and Duncan 1971[8] argued that the difference between these two stress states depends on the yield criterion and the shape of the corresponding yield locus. The effect of R on the FLDs depends on how the R-value affects the yield locus shape. Using (Hill's 1948) yield criterion, the stress ratios for positive strain ratios depend strongly on the value of R.

Hill's 1979 [9] yield criterion, taken with the assumption of the principle of equivalence of plastic work, was proposed to account for the so-called "anomalous behavior" of aluminum. This yield criterion has undergone application. One line of attack is represented by the work of Parmar and Mellor 1978[10].

Hosford 1979 [11] developed an extension of Hill's 1948 yield criterion , which is also found to be a special case of Hill's 1979 yield criterion. This criterion has been used by Graf and Hosford 1990[12] for sheet metals with normal anisotropy. Later, Padwal and Chaturvedi 1992[13] also used Hosford's 1985[14] planar anisotropy yield criterion to analyze the insatiable behavior of strain localization. They found that the effect of planar anisotropy is negligible while the predictions are strongly dependent on exponent "a" an exponent in yield criterion . Predictions with a=5,6 or 8 match the experimental results much better than the predictions that were obtained from Hill's yield criterion.

Friedman and Pan 2000 [15] studied the effect of different yield criteria of (Hill1948),(Hill1979)and(Hosford 1979) on the right hand side of the forming limit diagram .

Dariani and Azodi 2003[16] showed the agreement between theoretical and experimental results by changing the index of Hill1979 yield criterion for right and left hand of FLD.

Banabic2004 [17] determined the FLD of Aluminum alloy (Al-2008) using new yield criterion (BBC2000)[18] , showed the best agreement between theoretical and experiment result of right hand side of FLD.

In this paper the FLDs of different sheet metals: Steel, Brass and Aluminum alloy are obtained theoretically using, the Marciniak-Kuczynski Theory, the following yield criteria :Hill1948 , Hosford 1979 and modified Hosford 1985 [14] , These FLDs were compared with the experimentally obtained FLDs of the same sheets metals to obtain the best agreement between the calculated FLDs and the experimental FLDs.

2.Theoretical Analysis

The geometry of neck formation and the element of sheet undergoing plastic deformation are shown in Fig.1. Following the MK analysis , based on a simplified model with assumed pre-existing thickness imperfection in the form of a groove perpendicular to the principal strain directions **Fig.1**, The sheet is composed of the nominal area and weak groove area, which are denoted by 'a' and 'b', respectively. The initial imperfection factor of the groove, $f0$, is defined as the thickness ratio $f0$=(tbo/tao); where 't' denotes the thickness and subscript '0' denotes the initial state. A biaxial stress state is imposed on the nominal area and causes the development of strain increments in both the nominal (a) and the weak area (b).

The yield criterion proposed recently by modified Hosford was used in the calculation [14] in the plane stress state , this criterion is obtained as follows : $R_2(\sigma_1)^a + R_1(\sigma_2)^a + R_1R_2(\sigma_1 - \sigma_2)^a = R_2(R_1+1)\sigma'^a$ ....(1)

$$\sigma' = \left[\frac{1}{R_2(R_1+1)}\left(R_2(\sigma_1)^a + R_1(\sigma_2)^a + R_1R_2(\sigma_1-\sigma_2)^a\right)\right]^{\frac{1}{a}} \qquad (2)$$

$$\varphi = \frac{\sigma_1}{\sigma'} = \left[\frac{R_2(R_1+1)}{R_2 + R_1(\alpha)^a + R_1R_2(1-\alpha)^a}\right]^{\left(\frac{1}{a}\right)} \qquad (3)$$

The behavior of material can be represented in the form of Power law

$$\sigma' = K\varepsilon'^{n}\dot{\varepsilon}'^{m} \quad \text{.......... ....} (4)$$

The ratio of the principal stress and strain are define as follows:

$$\alpha = \frac{\sigma_2}{\sigma_1} \qquad ,\rho = \frac{\varepsilon_2}{\varepsilon_1} = \frac{d\varepsilon_2}{d\varepsilon_1} \text{..} (5)$$

The associated flow rule is expressed by

$$d\varepsilon_{ij} = d\lambda \frac{\partial \sigma'}{\partial \sigma_{ij}} \text{..} (6)$$

Thus, the yield criterion can be written as follows:

$$\frac{d\varepsilon_1}{R_2(\sigma_1)^{a-1} + R_1R_2(\sigma_1 - \sigma_2)^{a-1}} = \frac{d\varepsilon_2}{R_1(\sigma_2)^{a-1} - R_1R_2(\sigma_1 - \sigma_2)^{a-1}} = \frac{-d\varepsilon_3}{R_2(\sigma_1)^{a-1} + R_1(\sigma_2)^{a-1}} = \frac{d\varepsilon'}{R_2(R_1+1)\sigma'^{a-1}} \text{.........................} (7)$$

and

$$d\varepsilon_1 = \frac{d\varepsilon'\varphi^{a-1}}{R_2(R_1+1)}\left[R_2 + R_1R_2(1-\alpha)^{a-1}\right] \text{.......... ....} (8)$$

$$d\varepsilon_2 = \frac{d\varepsilon'\varphi^{a-1}}{R_2(R_1+1)}\left[R_1(\alpha)^{a-1} - R_1R_2(1-\alpha)^{a-1}\right] \text{.......... ....} (9)$$

from eq.(5)and(7)

$$\rho = \frac{d\varepsilon_2}{d\varepsilon_1} = \frac{R_1(\alpha)^{a-1} - R_1R_2(1-\alpha)^{a-1}}{R_2 + R_1R_2(1-\alpha)^{a-1}} \text{...} (10)$$

using condition of constant volume in plastic deformation

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0 \text{...} (11)$$

from eq.(8)and(9),(11)

$$d\varepsilon_3 = -\frac{d\varepsilon'\varphi^{a-1}}{R_2(R_1+1)}\left[R_2 + R_1(\alpha)^{a-1}\right] \ldots\ldots \quad (12)$$

then, by applying the principle of equivalence of plastic work

$$\sigma' d\varepsilon' = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 \ldots\ldots \quad (13)$$

$$\sigma' d\varepsilon' = \sigma_1 d\varepsilon_1 (1 + \alpha\rho) \ldots\ldots \quad (14)$$

$$\beta = \frac{d\varepsilon'}{d\varepsilon_1} = \varphi(1 + \alpha\rho) \ldots\ldots \quad (15)$$

the compatibility condition is given by

$$d\varepsilon_{2a} = d\varepsilon_{2b} \ldots\ldots \quad (16)$$

from Marciniak-Kuczynski analysis

$$f = \frac{t_b}{t_a} \ldots\ldots \quad (17)$$

$$f = f_o \exp(\varepsilon_{3b} - \varepsilon_{3a}) \ldots\ldots \quad (18)$$

the equilibrium condition requires that the applied load remains constant along the specimen ; therefore

$$F_{1a} = F_{1b} \ldots\ldots \quad (19)$$

$$\sigma_{1a} t_a = \sigma_{1b} t_b \ldots\ldots \quad (20)$$

from eq.(17)

$$\sigma_{1a} = f\sigma_{1b} \ldots\ldots \quad (21)$$

from eq.(3)

$$\varphi_a \sigma'_a = f\varphi_b \sigma'_b \ldots\ldots \quad (22)$$

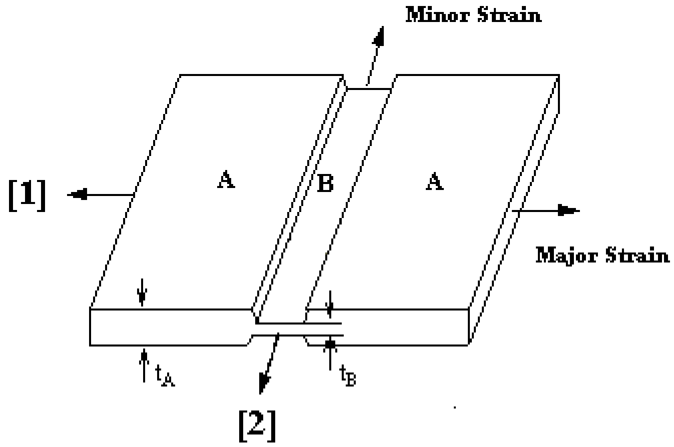
from eq.(4)

$$\varphi_a\left(\varepsilon'_a + d\varepsilon'_a\right)^n \dot{\varepsilon}'^{\,m}_a = f\varphi_b\left(\varepsilon'_b + d\varepsilon'_b\right)^n \dot{\varepsilon}'^{\,m}_b \;\ldots\ldots\ldots\ldots\;\ldots\ldots\ldots\ldots\;\ldots\ldots\;(23)$$

from eq.(18)

$$\varphi_a\left(\varepsilon'_a + d\varepsilon'_a\right)^n \dot{\varepsilon}'^{\,m}_a = f_o \exp\left(\varepsilon_{3b} - \varepsilon_{3a}\right)\varphi_b\left(\varepsilon'_b + d\varepsilon'_b\right)^n \dot{\varepsilon}'^{\,m}_b \;\ldots\ldots\ldots\;(24)$$

from eq.(10),(15) and (16)



$$\varphi_a\left(\varepsilon'_a + d\varepsilon'_a\right)^n \left(\frac{\beta_a}{\rho_a}\right)^m = f_o \exp\left(\varepsilon_{3b} - \varepsilon_{3a}\right)\varphi_b\left(\varepsilon'_b + d\varepsilon'_b\right)^n \left(\frac{\beta_b}{\rho_b}\right)^m \ldots(25)$$

Fig.(1) M-K analysis model[4]

Substitute equations(10),(12),(15) in equilibrium equation (25), an equation can be found and solved numerically. Imposing a loading path (ρa), a finite increment of strain is also imposed in region (a), and by numerical computation is performed by using computer program (Fortran power Station) to determine the limit strain of a strain path in the FLD , and the limit strain is determined when [(dεb/dεa)> 10] in the range of strain ratios from (-0.5 to 1.0).

3. Experimental Procedure

In the experimental study, mild Steel, brass and aluminum alloy sheets their chemical composition are shown in **table (1),(2)&(3)** were used .

Table(1) chemical analysis of Aluminum alloy

Material	Sn%	Ni%	Ti%	Cr%	Zn%	Mg%	Mn%	Cu%	Fe%	Si%	Al%
Aluminum alloy	0.001	0.0006	0.016	0.009	0.027	0.01	0.015	0.15	0.58	0.38	Rem.

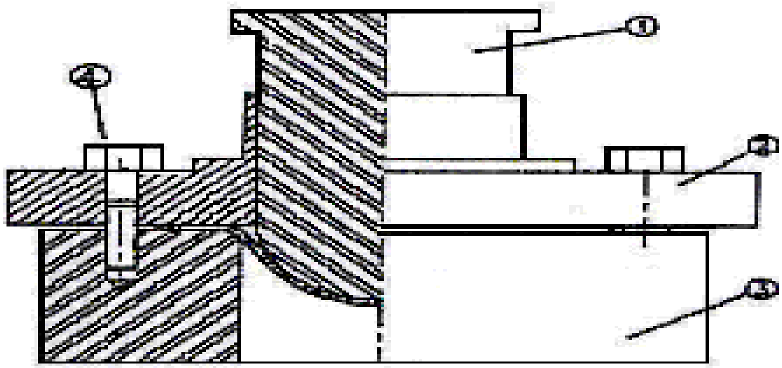
Table(2) chemical analysis of Mild Steel

Material	Cu%	Mo%	Ni%	Cr%	S%	P%	Si%	Mn%	C%	Fe%
Mild Steel	0.04	0.007	0.03	0.04	0.011	0.004	0.022	0.35	0.21	Rem.

Table(3) chemical analysis of α Brass

Material	Mn%	Pb%	Fe%	Zn%	Cu%
α Brass	0.001	0.001	0.19	26.2	Rem.

The FLDs of the sheets are determined using stretch forming tests with a hemispherical punch of (50mm) diameter and Die [19] with blank holder as shown in **Fig.2**.Using two sets of specimens with constant length (100mm) and having various widths with radius in one set for negative minor strain **Fig.3**.By changing the sheet width , major and minor strains were measured following varied deformation paths. Circular grids of (2mm) diameter were initially printed on the surface of the specimens for the purpose of strain measurements. For each specimen the strain were directly measured from deformed grids.



NO.	QTY	NAME	MAT.
1	1	PUNCH	STEEL
2	1	BLANKHOLDER	STEEL
3	1	DIE	STEEL
4	10	BOLT M17	STEEL

Fig.(2) Stretch forming test

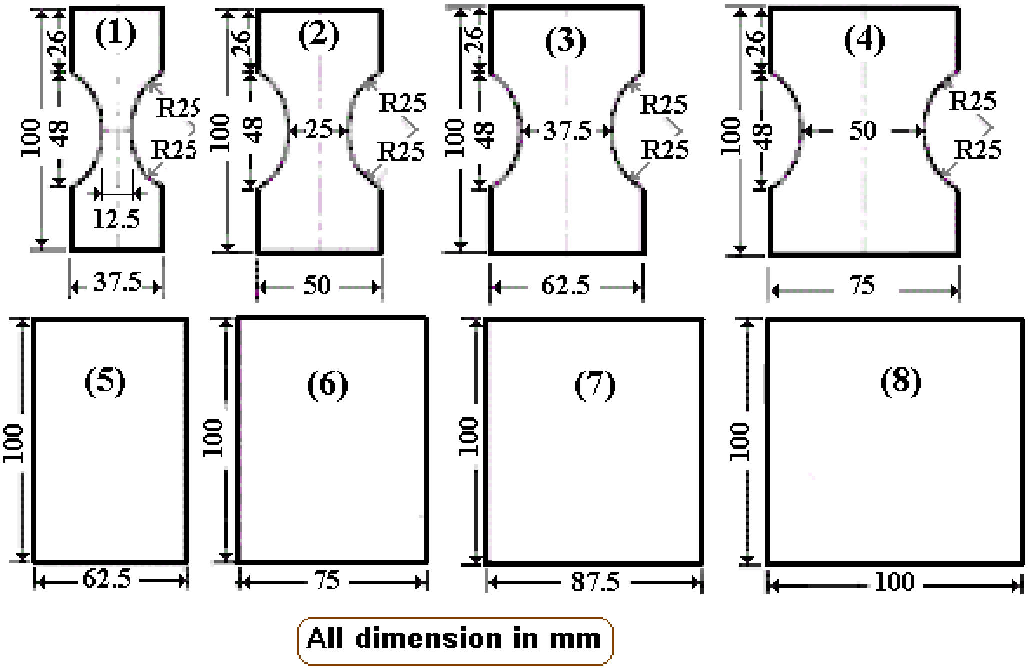


Fig.(3) Specimens used in the test

In the experimental work lubrication type(C140 Oil) was used .The FLD was obtained by drawing a line between the necked region and the un-necked (acceptable) region .

The mechanical properties of sheets metals were obtained from tensile test, by using specimens at different angles (٠,°٤٥°,٩٠°) to the rolling direction . After testing , the engineering stress-strain curve and true stress-strain curve were drawn. The value of strain hardening exponent (**n**) was determined from the slope of line in the (log coordinate of true stress strain curve) by selection two points one before ultimate stress and the other after yield point . The intersection of this line with unit strain gives the stress value that define the magnitude of strength coefficient (**K**) (**table 4**)[20] . For anisotropy plastic property (R) the same specimens of tensile test were used with different angles and using the equation **(1a)** and **(2a)** to determine the plastic anisotropic ratio**(table 4)** [20] .

$$R = \frac{\varepsilon_w}{\varepsilon_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

Where εw: strain in the width of specimen , εt: strain in the thickness of specimen

$$R' = \frac{R_0 + 2R_{45} + R_{90}}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

The strain rate sensitivity (m) was determined by using the same tensile test specimens , the cross-head speed is suddenly changed during the uniform deformation region of a tensile test and a small jump in the load may be observed , the exponent (m) is then calculated (**eq.3a**) from load and cross-head speed before and after the speed change , which is denoted by suffixes 1 and 2 in (**table 4)** [20].

$$m = \frac{\log\left(\frac{P_1}{P_2}\right)}{\log\left(\frac{V_1}{V_2}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

where P1: load before change , P2:load after change.

V1: cross head speed before change , V2: cross head speed after change

4.Results and Discussion

The values of **n,m,R′,R0,R90 and K** ,which were used in the theoretical determination of FLD, determined experimentally are shown in **table (4).**

Table (4) Properties of the used sheet materials

Material	**Strain Hardening exponent (n)**	**Strain rate sensitivity (m)**	**Normal plastic Anisotropic ratio (R′)**	**Strength coefficient (K)**	**Plastic anisotropic ratio with angle (0),(90)**	
					R0	**R90**
Aluminum alloy	0.2099	0.001	0.82608	272.5	1.1021	0.9012
Mild Steel	0.2607	0.016	1.4493	924.3	1.7511	1.5419
α- Brass	0.3215	0.005	0.9301	880.5	1.2214	0.9682

Fig.(4) show the experimental Forming limit curves of Aluminum alloy , Mild Steel and α-Brass as received , it is clear from the FLC for Brass is the highest while that for Aluminum alloy is the lowest , this ensure that the strain hardening exponent has the dominate effect on the formability of sheet metal .

Fig.(5),(6)&(7) show the comparison between the experimental and theoretical Forming limit Diagrams using (Hill 1948) yield criterion, It can be seen that this analysis overestimates the limit strain towards the equibiaxial strain path and underestimate the limit strains towards the plane strain and uniaxial regions ,

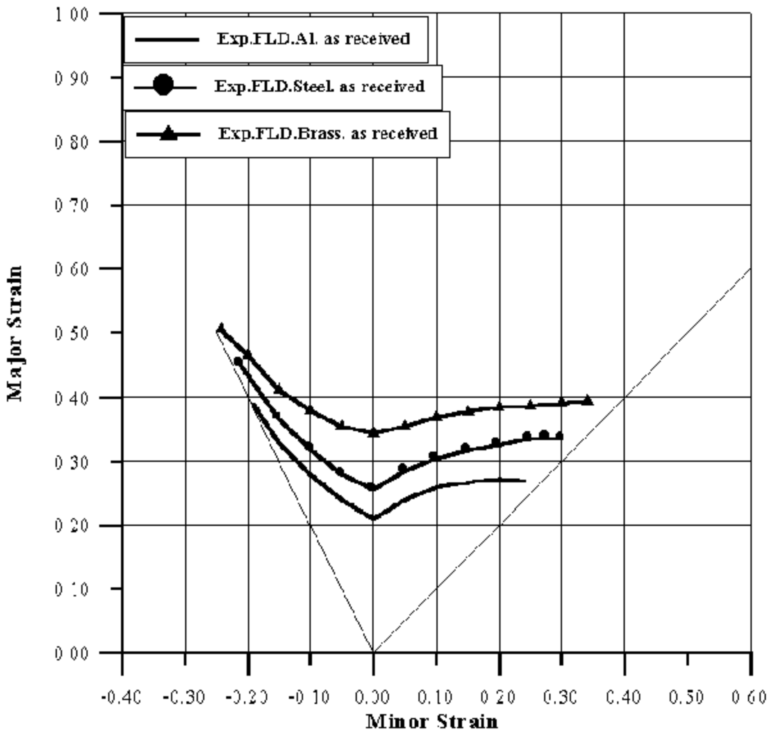


Fig.(4) Experimental FLDs of Aluminum alloy, Mild Steel & α- Brass

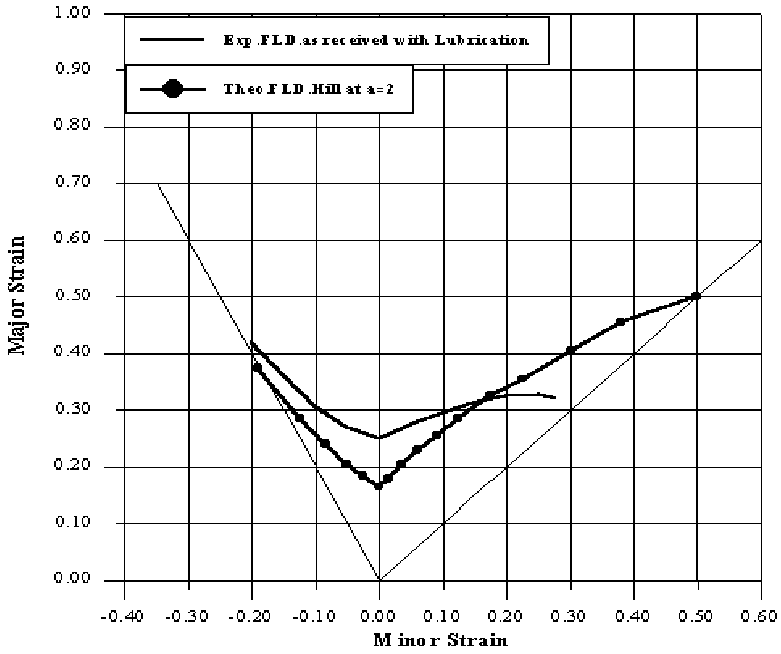


Fig.(5) Theoretical and experimental FLDs of Aluminum alloy

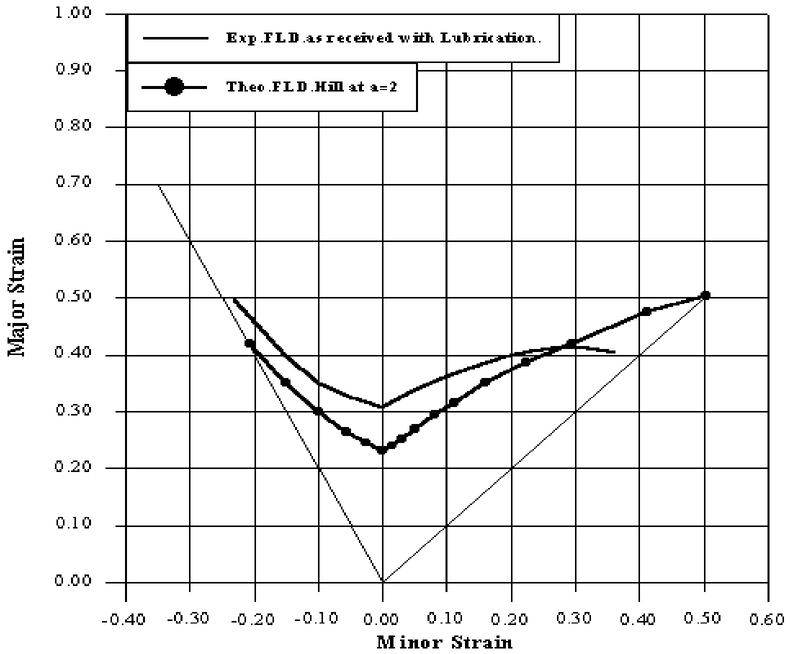


Fig.(6) Theoretical and experimental FLDs of Mild Steel

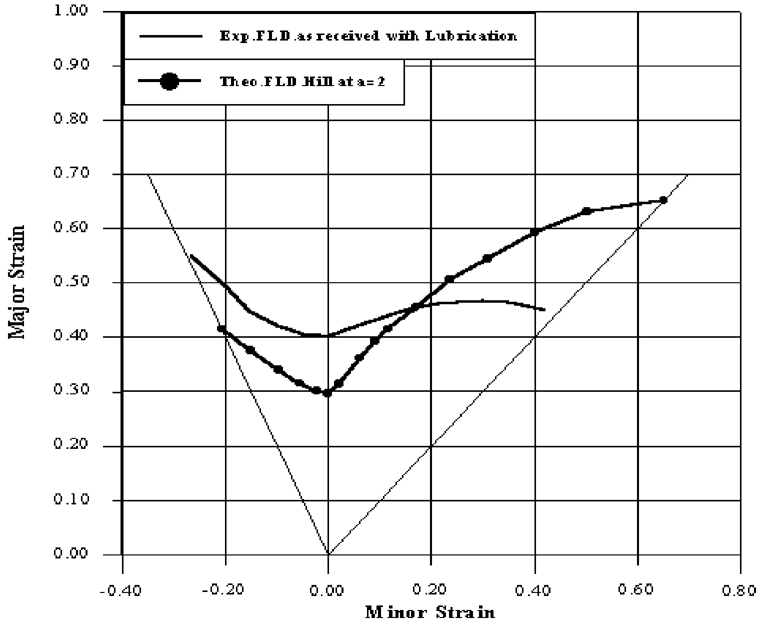


Fig.(7) Theoretical and experimental FLDs of α- Brass

Fig.(8),(9)&(10) Show the comparison of theoretical Forming limit Diagrams using Hosford 1979 yield criterion(exponent of yield criterion is 6 & 8) with experimental diagrams , It can be seen from comparison that the theoretical curves are closer to the experimental curves than these determined by Hill 1948 yield criterion .

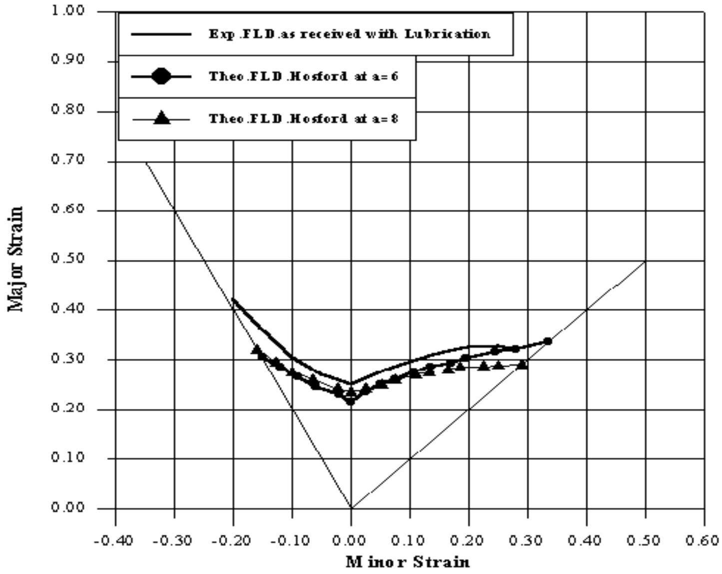


Fig.(8) Theoretical and experimental FLDs of Aluminum alloy

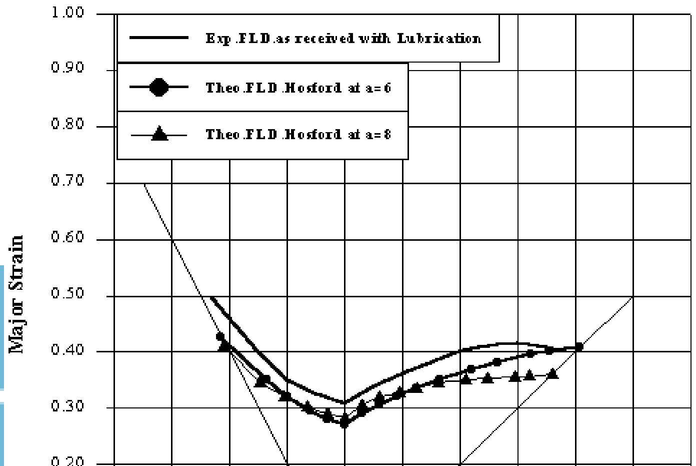


Fig.(9) Theoretical and experimental FLDs of Mild Steel

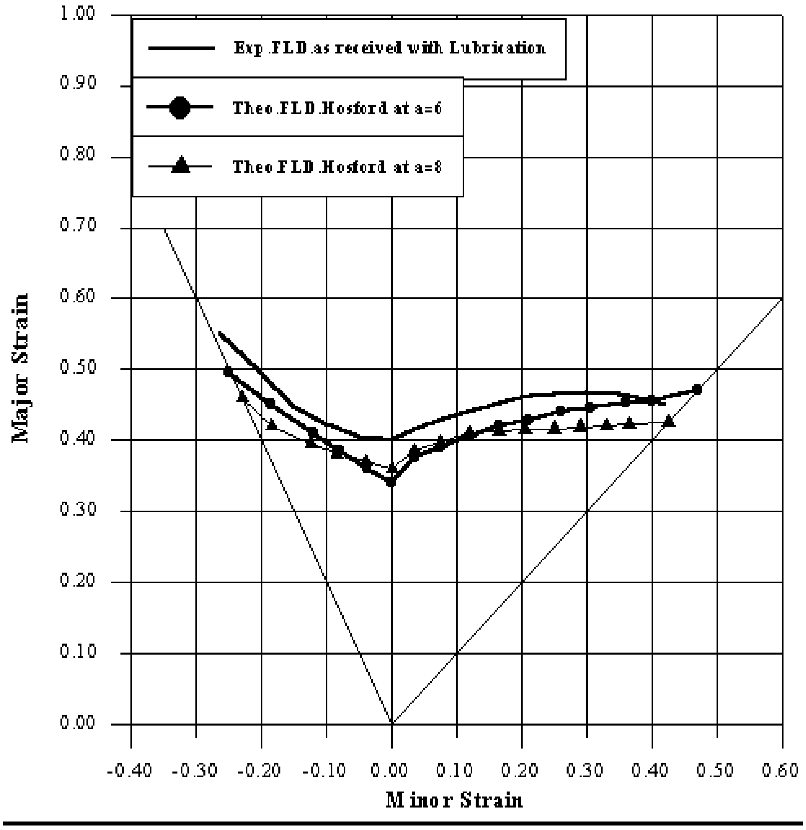


Fig.(10) Theoretical and experimental FLDs of α- Brass

Fig(11),(12)&(13) show that the determination of forming limit curves using modified Hosford 1985 criterion with the (M-K) analysis , gave the best results compared with the other used criteria .Using this criterion gave the closest forming limit curve to those obtained experimentally, but with different criterion index for different alloy ie, the value of the index (a=6) gave the best result for brass, while (a=8) gave the best results for aluminum alloy and mild steel.

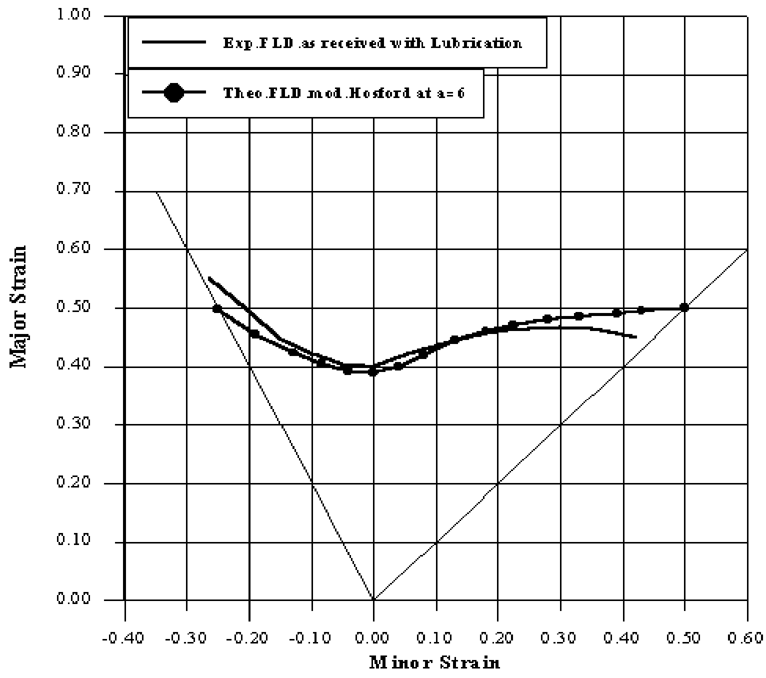


Fig.(11) Theoretical and experimental FLDs of α- Brass

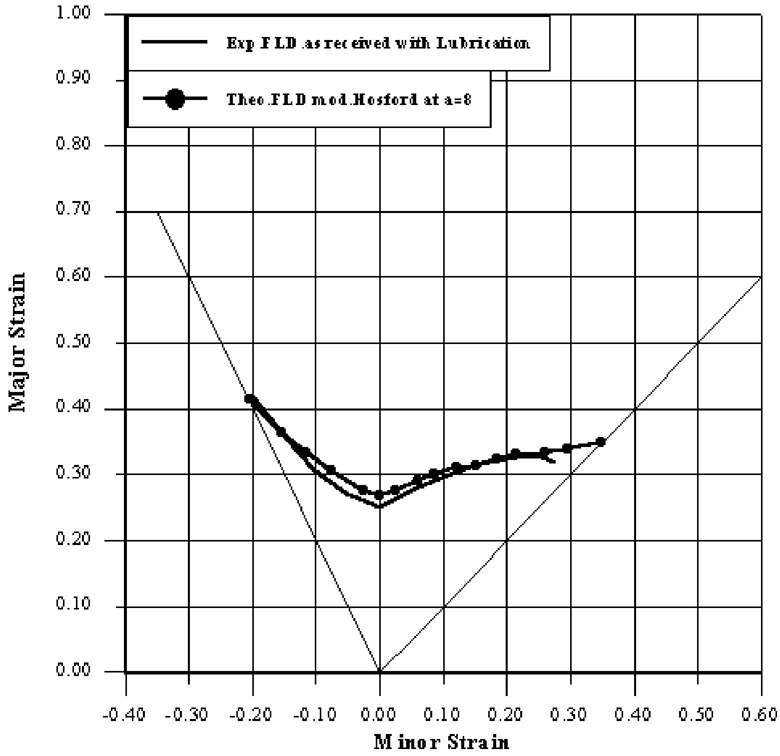


Fig.(12) Theoretical and experimental FLDs of Aluminum alloy

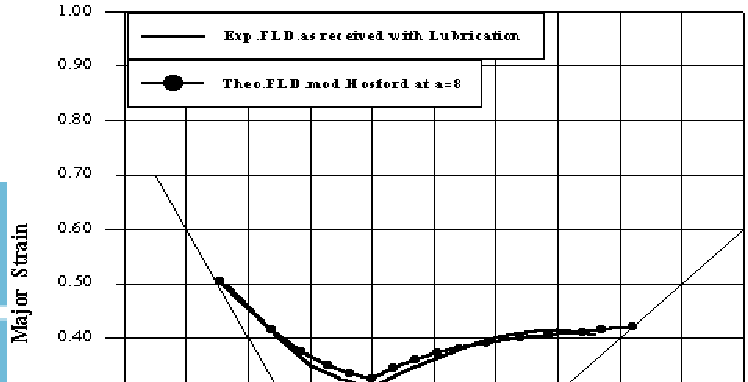


Fig.(13) Theoretical and experimental FLDs of Mild Steel

5.Conclusion

This work provides an experimental and theoretical analysis for the determination of the FLD using Hill1948, Hosford 1979 and modified Hosford 1985 yield criteria with Marciniak and Kuczynski (M-K) theory to find the best yield criterion (index of yield criterion) for the different materials, used the following conclusions can be obtained justified :

1.Efficiency of stretch forming by using hemispherical punch (50mm) and using eight type of specimens to cover the whole of forming limit curve .

2.Strain hardening exponent (n) has higher effect compared with other parameters this makes Brass has highest FLD while Aluminum alloy has the lowest.

3.In the theoretical results , it is shown that the determination of forming limit curve using modified Hosford 1985 criterion , gave the best results compared with the other used criteria , but with different index to different sheet metal.

4.The index (a=6)gave best result for Brass while the index (a=8)gave best result for Aluminum and mild Steel.

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